Laboratory work №4.6

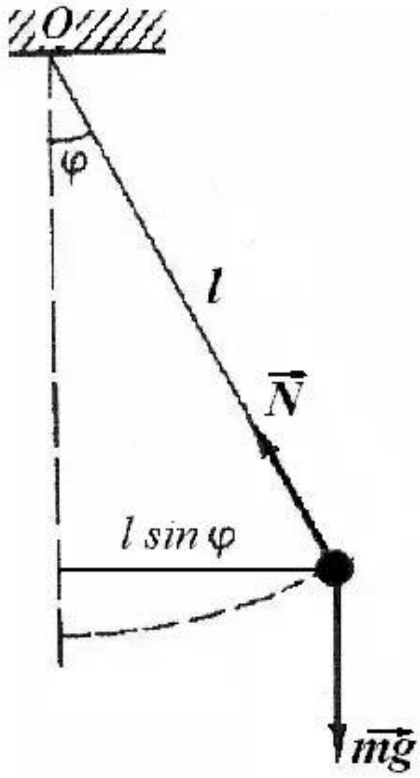
Investigation of free oscillations of mathematical pendulum

Tools: 1) mathematical pendulum; 2) stopwatch.

Purpose: to investigate oscillations of mathematical pendulum, to determine acceleration of free fall using pendulum.

Theory

A mathematical pendulum is an idealized system that consist from weightless, non-stretchable line on which a material point is suspended. Under the action of gravity force pendulum oscillates in the vertical plane.



A good approximation is a small heavy ball suspended on a long thin line.

We will characterize the deflection of the pendulum from the equilibrium position by the angle φ, that is formed by line and vertical.

According to the equation of the dynamics of rotational motion, the sum moment of external forces acting on the body is equal to the product of moment of inertia and angular acceleration

$$\sum M^{ext} = I\varepsilon.$$

Two forces act on pendulum – gravity force ***mg*** and tension force ***N***. When the pendulum deflects from the equilibrium position, a torque, created only by gravity force, arises. The moment of the tension force in respect to the point *O* equals zero because the force go through the suspension point *O*.

Torque *M* (moment of gravity force) equals to modulus of product of force ***mg*** to arm *l sin* φ (fig.)

$$M = -mgl\sin\varphi.$$

The minus is put because of the fact that the moment of force and the angular deflection have opposite signs.The angle φis counted counter clockwise and the force rotates clockwise.

Substitute inertia moment of material point $I = ml^2$ and angular acceleration like the second derivative from angle in respect to time$\varepsilon = \ddot{\varphi}$

$$ml^2\ddot{\varphi} = -mgl\sin\varphi. \ (1)$$

Let’s use the expansion of the sine in a Taylor series

$$\sin\varphi = \varphi - \frac{\varphi^3}{3!} + \frac{\varphi^5}{5!} - \cdots (2)$$

(the points denote the remaining terms of the series).

Hence we see that for sufficiently small φ we can neglect in (2) all terms of series except first.

Therefore, in the case of small oscillations, we can assume sine φ equal φ (in radians):

$$\sin\varphi = \varphi.$$

For example, for φ = 0,10rad (5,73°) sin φ= 0,0998,

$$0{,}1 \approx 0{,}0998.$$

For φ= 0,20rad (11,46°) sin φ= 0,1987

$$0{,}2 \approx 0{,}1987.$$

However forφ= 1,0rad (57,3°) sin φ= 0,841

$$1{,}0 \neq 0{,}841.$$

What angles correspond to "small" deflections? It depends on the accuracy of the measurements. If the count to two decimal digits, the angle φ should not exceed approximately 15°.

In connection with the foregoing, the equation of dynamics has the form

$$\ddot{\varphi} + \left(\frac{g}{l}\right)\varphi = 0.$$

Since the coefficient $\frac{g}{l}$ is positive, it can be designatedas the square of a several quantity

$$\omega^2 = \frac{g}{l}. \ (3)$$

As a result, we obtain the equation:

$$\ddot{\varphi} + \omega^2\varphi = 0. \ (4)$$

This equation is called differential, since it includes not only unknown quantity φ, but also its derivative (the second). A general method for solving such equations is considered in the course of higher mathematics. The solution (4) has the form

$$\varphi = a\ \cos(\omega t + \alpha_0). \ (5)$$

It is possible to verify by substitution in the differential equation that the solution satisfies it i.e. turns it into an identity.

Consequently in small oscillations, the angular deflection of a mathematical pendulum changes in time according to the harmonic law.

In this case, a - the absolute value of the largest angular displacement – called an amplitude, ω - angular frequencyof oscillations, $(\omega t + \alpha_0)$ – oscillations' phase, which determines the value of the displacement in the moment of time t, α_0 – initial phase.

The physical meaning of the angular frequency ω is related to the concept of the period T of oscillations. The period is the duration of one full wave i.e. the shortest period of time in which an arbitrarily chosen state of the oscillatory system repeats. During one period of the oscillation phase receive increment 2π

$$\omega(t + T) + \alpha_0 = \omega t + \alpha_0 + 2\pi,$$

hence

$$T = \frac{2\pi}{\omega}. \ (6)$$

Then, taking (3) into account, we obtain the formula of period of the mathematical pendulum's oscillations

$$T = 2\pi\sqrt{\frac{l}{g}}. \ (7)$$

From the formula of oscillations' period follow such regularities of mathematical pendulum's oscillations:

1) the period of pendulum's oscillations doesn't depend on the amplitude of the oscillations (for small deflection angle);
2) the period of pendulum's oscillations doesn't depend on the mass of the pendulum;
3) the period of pendulum's oscillations is directly proportional to the square root of the length of the pendulum and inversely proportional to the square root of the acceleration of free fall.

A mathematical pendulum is used for measuring the acceleration of free fall. From formula (7) follows that

$$g = \frac{4\pi^2}{T^2}l. \ (8)$$

With the movement from the pole to the Earth's equator, the acceleration of free fall due to the rotation of the Earth decreases from the values $g = 9{,}83\ \frac{\text{м}}{\text{с}^2}$ at the pole to $g = 9{,}78\frac{\text{м}}{\text{с}^2}$ at the equator.

However, the Earth crust in different places has a different composition; therefore, in places where the crust has a higher density, the acceleration of free fall increases. By changing of g in a certain area, measuring it with a mathematical pendulum, geologists judge changes the Earth crust's density and then using these data conclude about existence of minerals. This is the so-called gravitational exploration of minerals, which is used in geophysics.

Measurement

The work consists of two parts.

1. **Ascertainment of oscillations' *isochronisms*** i.e. independence of the oscillations' period from the amplitude.

According to theory the angle of deflection is small. The table shows the different values of the deflections, corresponding angels in degrees and radians, and the sines of these small angles

x	*angleφ, degrees*	sin φ	*angleφ,radians*
20 см	5, 73	0,10	0,10
30 см	8,63	0,15	0,15
40 см	11,54	0,20	0,20

As we can see from the table, the requirement of equality of the sine of the angle φ to the angle φ itself, expressed in radians, with these deflections is satisfied (accuracy to two decimal digits), therefore, the angles can be considered as small for a given line length l = 2.95 m.

Using a stopwatch, determine the period of oscillations of the pendulum for various initial deflection of the pendulum from the equilibrium position. The experiment is performed for x = 20, 30 and 40 cm. Each time, we determine the total time of a large number of oscillations (30 ± 50 full, i.e., back and forth oscillations) and the oscillation period of the pendulum. The period is determined three times for each initial deflection.

Record the data of experiment. We are convinced that the period of oscillations does not depend on the amplitude of the oscillations (the initial deflection of the pendulum).

2. **Calculation of the *acceleration of free fall* g.**

According to the measurements of the periods recorded in the table, using formula (8), determine the acceleration of free fall for a given locality.

Then, using the approximate formula for the dependence of the acceleration of free fall on the geographical latitude of the locality

$$g = 9{,}78049\,(1 + 0{,}0052884\,\sin^2\varphi_{lat} - 0{,}0000059\,\sin^2 2\varphi_{lat}) - 0{,}00011$$

calculate the theoretical value of g for the latitude of Dnepropetrovsk (φ_{lat} = 48°27 'north latitude) and compare it with the value obtained in the experiment.

Control questions

1. Substitute solution (5) into equation (4) and make sure that this expression turns it into an identity. Under what conditions is this possible?
2. List the regularities of harmonic pendulum’s oscillations.
3. At what angles can the pendulum oscillations be considered harmonic and why?

x	l	T_i	$<T>$	ΔT_i	$S_{<T>}$	ΔT	$E,\%$	g_i	$<g>$	Δg_i	$S_{<g>}$	Δg	$E,\%$

$$g = <g> \pm \Delta g$$